

GABARITO - G.A.A.L. 2º Bim. (2010/11)
Física Max

① $\vec{n} = \vec{a} \times \vec{b} = (0, 2, -1)$

② $\vec{s} = -\vec{BC} = -(C-B) = -[(-2, -1, 3) - (2, 3, 0)]$ (1)

$\vec{s} = -(-4, -4, 3) = (4, 4, -3)$

$A = \|\vec{n} \times \vec{s}\|$

$A = 9,17 \mu^2$

$\vec{n} \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ 4 & 4 & -3 \end{vmatrix}$

$\vec{n} \times \vec{s} = -6\vec{i} - 4\vec{j} - 9\vec{k} + 4\vec{j} = -6\vec{i} - 4\vec{j} - 9\vec{k}$

$\vec{n} \times \vec{s} = (-6, -4, -9)$

$\|\vec{n} \times \vec{s}\| = \sqrt{x^2 + y^2 + z^2} = \sqrt{36 + 16 + 81} = \sqrt{133}$

$\|\vec{n}\| = \sqrt{0^2 + 2^2 + 1^2} = \sqrt{5}$

② $\angle(B) = \angle(\vec{a}, \vec{b}) = \theta$ (*) $\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$

$\vec{a} = \vec{BA} = A - B = (1, 0, -2) - (2, 3, 0)$

$\vec{a} = (-1, -3, -2)$

$\vec{b} = \vec{BC} = C - B = (-2, -1, 3) - (2, 3, 0)$

$\vec{b} = (-4, -4, 3)$

$\vec{a} \cdot \vec{b} = 4 + 12 - 6 = 10$

$\|\vec{a}\| = \sqrt{(-1)^2 + (-3)^2 + (-2)^2} = \sqrt{14}$

$\|\vec{b}\| = \sqrt{(-4)^2 + (-4)^2 + 3^2} = \sqrt{41}$

SUBST. EM (*): $\cos(\theta) = \frac{10}{\sqrt{14} \cdot \sqrt{41}} = \frac{10}{\sqrt{574}} \Rightarrow \theta \approx 65,33^\circ$

④ $\vec{n} \perp \vec{u} \Rightarrow \vec{n} \cdot \vec{u} = 0$

$\vec{n} = (a, b, c)$

$\vec{u} = (0, 1, -2)$

$\vec{s} = (3, 2, 0)$

$(0, 1, -2) \cdot (a, b, c) = 0$

$b - 2c = 0 \Rightarrow b = 2c$

$\vec{n} = (1, 2, 1) = \vec{s}$ (ou outro)

(*) $\angle(\vec{n}, \vec{s}) = \angle(\vec{n}, \vec{s}) = \theta$

$\cos(\theta) = \frac{|\vec{n} \cdot \vec{s}|}{\|\vec{n}\| \cdot \|\vec{s}\|}$

$\vec{n} \cdot \vec{s} = (1, 2, 1) \cdot (3, 2, 0) = 3 + 6 = 9$

$\|\vec{n}\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\|\vec{s}\| = \sqrt{3^2 + 2^2 + 0^2} = \sqrt{13}$

CONT. 4

$$\cos(\theta) = \frac{|13|}{\sqrt{36} \cdot \sqrt{13}} = \frac{13}{\sqrt{208}} \Rightarrow \theta \approx 25.66^\circ$$

③ VPS



$$\vec{N} = \vec{BC} \text{ ou } \vec{CB}$$

Pontos: B ou C.

i) $\vec{N} = \vec{BC} = C - B = (-4, 4, 3)$ (exat 1 e 2)

$$\vec{N} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix} \text{ e } B = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

a) Eq. Vetr.

$$(x, y, z) = (x_0 + at, y_0 + bt, z_0 + ct) \quad t \in \mathbb{R} \quad (1.5)$$

$$(x, y, z) = (2 - 4t, 3 + 4t, 0 + 3t)$$

c) Eq. Param.

$$\begin{cases} x = 2 - 4t \\ y = 3 + 4t \\ z = 3t \end{cases}$$

d) Eq. Simétricas

$$\frac{x-2}{-4} = \frac{y-3}{4} = \frac{z}{3} \quad \text{ou}$$

$$\frac{2-x}{4} = \frac{3-y}{4} = \frac{z}{3}$$

⑤ Plano θ

$$A = (1, 0, -2)$$

$$\vec{A} = \vec{n} = (3, 2, 0)$$

i) $ax + by + cz + d = 0$

$$3x + 2y + 0z - 3 = 0$$

$$3x + 2y - 3 = 0$$

$$d = -(\vec{A} \cdot \vec{n})$$

$$d = -(3 + 0 + 0) \Rightarrow d = -3$$



⑥ $d(\pi, \lambda)$

$$A \in \lambda \quad (t=0) \quad \lambda = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}$$

$$\pi: -2x + 3z - 4 = 0$$

$$\lambda: (x, y, z) = (3t+1, 2t, -4)$$

$$(*) \quad d(\pi, \lambda) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d(\pi, \lambda) = \frac{|-2(1) + 0(0) + 3(-4) - 4|}{\sqrt{(-2)^2 + 0^2 + 3^2}} = \frac{-18}{\sqrt{13}} = \frac{18}{\sqrt{13}}$$

$$d(\pi, \lambda) \approx 4.99 \mu$$

⑦
 (i) $P, Q \in \Lambda$ (exerc. 4) $\Lambda \Rightarrow (x, y, z) = (3t+1, 2t, -4)$

$P(t=1) \Rightarrow (x, y, z) = (4, 2, -4)$

$Q(t=2) \Rightarrow (x, y, z) = (7, 4, -4)$

~~Q, P~~
 JP

(ii) $L, M \in \Theta$ (exerc. 6)

$\Theta = \Pi$ $\Pi: -2x + 3z - 4 = 0$

$L = (1, 0, 3)$

$-2(1) + 3(3) - 4 = 0$

$L = (1, 0, 2)$

$3z = 4 + 2 \Rightarrow z = \frac{6}{3} \Rightarrow z = 2$

$M = (2, 1, 3)$

$-2 \cdot (2) + 3(3) - 4 = 0$

$M = (2, 1, 8/3)$

$-4 + 3z = 4 \Rightarrow z = \frac{4+4}{3} \Rightarrow \boxed{8/3 = z}$