

(GABARITO - C.I. - 2º BIM. / 23/06/2026)
(UFR) - Prof. PAULO ALEX. (ENG. DE ALIM.)

a) $g(x) = 2x^3 - \lg(x) - \frac{1}{4x^{-2}} + 3$
 $g(x) = 2x^3 - \lg(x) - \frac{1}{4}x^2 + 3$ (MUT. B.S.)
 $g'(x) = 2 \cdot 3x^{3-1} - \sec^2(x) - \frac{1}{4} \cdot 2x^{2-1} + 0$
 $g'(x) = 6x^2 - \sec^2(x) - \frac{1}{2}x$

$\frac{1}{a-1} = a, \frac{1}{x^{-2}} = x^{-(-2)} = x^2$
 $f(x) = ax^n \Rightarrow f'(x) = a \cdot n \cdot x^{n-1}$
 $f(x) = \lg(x) \Rightarrow f'(x) = \sec^2(x)$
 $f(x) = c \Rightarrow f'(x) = 0$

b) $p(x) = \sec(3x+1) \cdot (x^2-4x)$

(*) $p(x) = a(x) \cdot b(x) \Rightarrow p'(x) = a'(x) \cdot b(x) + a(x) \cdot b'(x)$

ONDE: $a(x) = \sec(3x+1) \Rightarrow a'(x) = \cos(3x+1) \cdot 3$ (R. DA DERIVADA)

$b(x) = (x^2-4x) \Rightarrow b'(x) = 2x-4$

SUBST. EM (*):

$p'(x) = 3\cos(3x+1) \cdot (x^2-4x) + \sec(3x+1) \cdot (2x-4)$

$p'(x) = (3x^2-12x)\cos(3x+1) + (2x-4)\sec(3x+1)$

c) $r(x) = \ln(45x^{-2}) - 3/x + \sqrt{x^3+3x}$

(*) $r(x) = f(x) + p(x) + n(x)$, ONDE:

① $f(x) = \ln(45x^{-2}) \Rightarrow f'(x) = \frac{1}{45x^{-2}} \cdot (-2(45)x^{-3}) = -\frac{2}{x}$
 $f'(x) = -2x^{-1}$

② $p(x) = -3/x = -3x^{-1}$
 $p'(x) = -3(-1)x^{-1-1} = 3x^{-2}$

③ $n(x) = \sqrt{x^3+3x}$ (R. DA DERIVADA)
 $n'(x) = \frac{1}{2\sqrt{x^3+3x}} \cdot (3x^2+3) = \frac{3x^2+3}{2\sqrt{x^3+3x}}$

SUBST. EM (*):

$r'(x) = f'(x) + p'(x) + n'(x)$

$r'(x) = -\frac{2}{x} + \frac{3}{x^2} + \frac{3x^2+3}{2\sqrt{x^3+3x}} = -\frac{2}{x} + \frac{3}{x^2} + \frac{3x^2+3}{2\sqrt{x^3+3x}}$

d) $h(x) = \frac{(x^2+2x)^3}{\cos(3x)-5x^3} = \frac{a(x)}{b(x)} \Rightarrow$ (i) $h'(x) = \frac{a'(x)b(x) - a(x)b'(x)}{[b(x)]^2}$

ONDE ① $a(x) = (x^2+2x)^3 \Rightarrow a'(x) = 3(x^2+2x)^2 \cdot (2x+2)$

② $b(x) = \cos(3x)-5x^3 \Rightarrow b'(x) = -\sin(3x) \cdot 3 - 15x^2 = -3(\sin(3x)+5x^2)$

SUBST. EM (*): $h'(x) = \frac{[3(x^2+2x)^2(2x+2)] \cdot [\cos(3x)-5x^3] - [(x^2+2x)^3] \cdot [-3(\sin(3x)+5x^2)]}{[\cos(3x)-5x^3]^2}$

1.d) (CONTINUAR)

$$h'(x) = \frac{6(1-x^{-3}) \cdot (x^{-2}+2x)^3 (\cos(3x) - 5x^3) + 3 \cdot (x^{-2}+2x)^3 (5x^2 + \sin(3x))}{[\cos(3x) - 5x^3]^2}$$

2) Eq da reta tang. (*) $y - y_0 = m(x - x_0)$; $m = f'(x_0)$ (1,25)

$$f(x) = \sqrt[3]{6x+2} = (6x+2)^{\frac{1}{3}}$$

1) $f'(x) = \frac{1}{3}(6x+2)^{-\frac{2}{3}} \cdot 6$

(i) $x_0 = 1$ $y_0 = f(x_0) = f(1)$

$$y_0 = \sqrt[3]{6 \cdot (1) + 2} = \sqrt[3]{8} = 2 \Rightarrow y_0 = 2$$

$$f'(x) = \frac{6}{3 \sqrt[3]{(6x+2)^2}} = \frac{2}{\sqrt[3]{(6x+2)^2}}$$

(ii) $m = f'(x_0) = f'(1)$

$$m = \frac{2}{\sqrt[3]{(6 \cdot (1) + 2)^2}} = \frac{2}{\sqrt[3]{8^2}} = \frac{2}{4} = \frac{1}{2} = m$$

SUBST. em (*):

$$y - 2 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x - \frac{1}{2} + 2 \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$$

3) $g(y) = f^{-1}(x)$ (*) $g'(y) = \frac{1}{f'(x)}$

$$y = f(x)$$

SUBST. em (*):

(i) $f(x) = \sin(x) \Rightarrow f'(x) = \cos(x)$

$$g'(0.5) = \frac{1}{f'(\frac{\pi}{6})}$$

(ii) $0.5 = \sin(x) \Rightarrow$ Pela calc $\Rightarrow x = \frac{\pi}{6}$

(iii) $f'(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ (TABELA)

$$g'(0.5) = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

4) $3y + x^2 + e^{2y} = x^2y^2 + 5x$ $y(x)$

$$3y' + 2x + e^{2y} \cdot (2) \cdot y' = (2x \cdot y^2 + x^2 \cdot 2y \cdot y') + 5$$

$$3y' + 2e^{2y}y' - 2x^2yy' = 5 - 2x + 2xy^2 \Rightarrow y' = \frac{2xy^2 + 5 - 2x}{3 + 2e^{2y} - 2x^2y}$$

$$y'(3 + 2e^{2y} - 2x^2y) = 2xy^2 + 5 - 2x$$

5) $f(x) = \frac{1}{3}x^3 - 9x + 4$

(i) P. CRÍTICOS. $f'(x) = 0$

$$f'(x) = x^2 - 9 = 0 \Rightarrow (x+3) \cdot (x-3) = 0$$

$$x' = 3 \text{ e } x'' = -3$$

(iii) P. MÍN. $f'(x) = 0$ e $f''(x) > 0$

$$f''(x) = 2x \Rightarrow f''(3) = 6 > 0$$

$$x = 3$$

Logo, $x = 3$ é o P. MÍN., e $x = -3$ é o

(ii) P. MÁX. $f'(x) = 0$ e $f''(x) < 0$

$$f''(x) = 2x \quad x = -3$$

$$f''(-3) = -6 < 0$$

Ponto Máx. = $(-3, 14)$; Ponto Mín. = $(3, -14)$

$$\textcircled{6} \quad \int x^2 + 2x - \text{SEN}(x) \, dx = A(x)$$

$$(*) \quad \int ax^n \, dx = \frac{a \cdot x^{n+1}}{n+1} + C ; \quad \int \text{SEN}(x) \, dx = \cos(x) + C$$

$$A(x) = \frac{x^3}{3} + \frac{2x^2}{2} + \cos(x) + C$$

$$A(x) = \frac{x^3}{3} + x^2 + \cos(x) + C$$

$$\textcircled{b) \quad \int_2^3 \frac{1}{2}x + \frac{1}{x} \, dx = I$$

$$(*) \quad \int_a^b f(x) \, dx = F(x) \Big|_a^b = f(b) - f(a) ; \quad F'(x) = f(x)$$

$$I = \left[\frac{1}{2} \frac{x^2}{2} + \ln|x| \right]_2^3 = \left[\frac{x^2}{4} + \ln|x| \right]_2^3$$

$$I = \left[\frac{9}{4} + \ln(3) \right] - \left[\frac{4}{4} + \ln(2) \right] \quad (\text{CALCULADORA})$$

$$\textcircled{c} \quad I \approx 1,66 \quad \Rightarrow \quad \int_2^3 \frac{1}{2}x + \frac{1}{x} \, dx \approx 1,66$$