

## TABELA DE DERIVADAS

1.  $D_x(u^n) = nu^{n-1} D_x u$
2.  $D_x(u + v) = D_x u + D_x v$
3.  $D_x(uv) = u D_x v + v D_x u$
4.  $D_x\left(\frac{u}{v}\right) = \frac{v D_x u - u D_x v}{v^2}$
5.  $D_x(e^u) = e^u D_x u$
6.  $D_x(a^u) = a^u \ln a D_x u$
7.  $D_x(\ln u) = \frac{1}{u} D_x u$
8.  $D_x(\sin u) = \cos u D_x u$
9.  $D_x(\cos u) = -\sin u D_x u$
10.  $D_x(\operatorname{tg} u) = \sec^2 u D_x u$
11.  $D_x(\operatorname{cotg} u) = -\operatorname{cosec}^2 u D_x u$
12.  $D_x(\sec u) = \sec u \operatorname{tg} u D_x u$
13.  $D_x(\operatorname{cosec} u) = -\operatorname{cosec} u \operatorname{cotg} u D_x u$
14.  $D_x(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} D_x u$
15.  $D_x(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} D_x u$
16.  $D_x(\operatorname{tg}^{-1} u) = \frac{1}{1+u^2} D_x u$
17.  $D_x(\operatorname{cotg}^{-1} u) = \frac{-1}{1+u^2} D_x u$
18.  $D_x(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} D_x u$
19.  $D_x(\operatorname{cosec}^{-1} u) = \frac{-1}{u\sqrt{u^2-1}} D_x u$
20.  $D_x(\sinh u) = \cosh u D_x u$
21.  $D_x(\cosh u) = \sinh u D_x u$
22.  $D_x(\operatorname{tgh} u) = \operatorname{sech}^2 u D_x u$
23.  $D_x(\operatorname{cotgh} u) = -\operatorname{cosech}^2 u D_x u$
24.  $D_x(\operatorname{sech} u) = -\operatorname{sech} u \operatorname{tgh} u D_x u$
25.  $D_x(\operatorname{cosech} u) = -\operatorname{cosech} u \operatorname{cotg} u D_x u$

## TABELA DE INTEGRAIS

## Algumas Formas Elementares

1.  $\int du = u + C$
2.  $\int a du = au + C$
3.  $\int [f(u) + g(u)] du = \int f(u) du + \int g(u) du$
4.  $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$
5.  $\int \frac{du}{u} = \ln|u| + C$

Formas Racionais Contendo  $\alpha + bu$ 

6.  $\int \frac{u du}{a + bu} = \frac{1}{b^2} [a + bu - a \ln|a + bu|] + C$
7.  $\int \frac{u^2 du}{a + bu} = \frac{1}{b^3} \left[ \frac{1}{2}(a + bu)^2 - 2a(a + bu) + a^2 \ln|a + bu| \right] + C$
8.  $\int \frac{u du}{(a + bu)^2} = \frac{1}{b^2} \left[ \frac{a}{a + bu} + \ln|a + bu| \right] + C$
9.  $\int \frac{u^2 du}{(a + bu)^2} = \frac{1}{b^3} \left[ a + bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right] + C$
10.  $\int \frac{u du}{(a + bu)^3} = \frac{1}{b^2} \left[ \frac{a}{2(a + bu)^2} - \frac{1}{a + bu} \right] + C$
11.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
12.  $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
13.  $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} + \frac{1}{a^2} \ln \left| \frac{u}{a + bu} \right| + C$

Formas Contendo  $\sqrt{a + bu}$ 

14.  $\int u \sqrt{a + bu} du = \frac{2}{15b^3} (3bu - 2a)(a + bu)^{3/2} + C$
15.  $\int u^2 \sqrt{a + bu} du = \frac{2}{105b^3} (15b^2u^2 - 12abu + 8a^2)(a + bu)^{3/2} + C$
16.  $\int u^n \sqrt{a + bu} du = \frac{2u^n(a + bu)^{3/2}}{b(2n + 3)} - \frac{2an}{b(2n + 3)} \int u^{n-1} \sqrt{a + bu} du$
17.  $\int \frac{u du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$
18.  $\int \frac{u^2 du}{\sqrt{a + bu}} = \frac{2}{15b^3} (3b^2u^2 - 4abu + 8a^2) \sqrt{a + bu} + C$
19.  $\int \frac{u^n du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2an}{b(2n + 1)} \int \frac{u^{n-1} du}{\sqrt{a + bu}}$

$$20. \int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C & \text{se } a > 0 \\ \frac{2}{\sqrt{-a}} \operatorname{tg}^{-1} \sqrt{\frac{a+bu}{-a}} + C & \text{se } a < 0 \end{cases}$$

$$21. \int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{a(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}}$$

$$22. \int \frac{\sqrt{a+bu} du}{u} = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

$$23. \int \frac{\sqrt{a+bu} du}{u^n} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)} \int \frac{\sqrt{a+bu} du}{u^{n-1}}$$

### Formas Contendo $\alpha^2 \pm u^2$

$$24. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \operatorname{tg}^{-1} \frac{u}{a} + C$$

$$25. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C = \begin{cases} \frac{1}{a} \operatorname{tgh}^{-1} \frac{u}{a} + C & \text{se } |u| < a \\ \frac{1}{a} \operatorname{cotg}^{-1} \frac{u}{a} + C & \text{se } |u| > a \end{cases}$$

$$26. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C = \begin{cases} -\frac{1}{a} \operatorname{tgh}^{-1} \frac{u}{a} + C & \text{se } |u| < a \\ -\frac{1}{a} \operatorname{cotg}^{-1} \frac{u}{a} + C & \text{se } |u| > a \end{cases}$$

### Formas Contendo $\sqrt{u^2 \pm \alpha^2}$

Nas fórmulas de 27 a 38, podemos substituir

$\ln(u + \sqrt{u^2 + a^2})$  por  $\operatorname{senh}^{-1} \frac{u}{a}$

$\ln|u + \sqrt{u^2 - a^2}|$  por  $\operatorname{cosh}^{-1} \frac{u}{a}$

$\ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right|$  por  $\operatorname{senh}^{-1} \frac{a}{u}$

$$27. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$28. \int \sqrt{u^2 \pm a^2} du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$29. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{u}{8} (2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$30. \int \frac{\sqrt{u^2 + a^2} du}{u} = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$31. \int \frac{\sqrt{u^2 - a^2} du}{u} = \sqrt{u^2 - a^2} - a \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$32. \int \frac{\sqrt{u^2 \pm a^2} du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$33. \int \frac{u^2 du}{\sqrt{u^2 \pm a^2}} = \frac{u}{2} \sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$34. \int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$35. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \frac{u}{a} + C$$

$$36. \int \frac{du}{u^2 \sqrt{u^2 \pm a^2}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$$

$$37. \int (u^2 \pm a^2)^{3/2} du = \frac{u}{8} (2u^2 \pm 5a^2) \sqrt{u^2 \pm a^2} + \frac{3a^4}{8} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$38. \int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{u}{\pm a^2 \sqrt{u^2 \pm a^2}} + C$$

### Formas Contendo $\sqrt{a^2 - u^2}$

$$39. \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$40. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$41. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$42. \int \frac{\sqrt{a^2 - u^2} du}{u} = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$= \sqrt{a^2 - u^2} - a \operatorname{cosh}^{-1} \frac{a}{u} + C$$

$$43. \int \frac{\sqrt{a^2 - u^2} du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$44. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$45. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C \\ = -\frac{1}{a} \operatorname{cosh}^{-1} \frac{a}{u} + C$$

$$46. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$47. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$48. \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

### Formas Contendo $2au - u^2$

$$49. \int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$50. \int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} \\ + \frac{a^3}{2} \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$51. \int \frac{\sqrt{2au - u^2} du}{u} = \sqrt{2au - u^2} + a \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$52. \int \frac{\sqrt{2au - u^2} du}{u^2} = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$53. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$54. \int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$55. \int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)\sqrt{2au - u^2}}{2} + \frac{3a^2}{2} \cos^{-1} \left( 1 - \frac{u}{a} \right) + C$$

$$56. \int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$

$$57. \int \frac{du}{(2au - u^2)^{3/2}} = \frac{u-a}{a^2 \sqrt{2au - u^2}} + C$$

$$58. \int \frac{u du}{(2au - u^2)^{3/2}} = \frac{u}{a \sqrt{2au - u^2}} + C$$

### Formas Contendo Funções Trigonométricas

$$59. \int \operatorname{sen} u du = -\cos u + C$$

$$60. \int \cos u du = \operatorname{sen} u + C$$

$$61. \int \operatorname{tg} u du = \ln |\sec u| + C$$

$$62. \int \operatorname{cotg} u du = \ln |\operatorname{sen} u| + C$$

$$63. \int \sec u du = \ln |\sec u + \operatorname{tg} u| + C = \ln \left| \operatorname{tg} \left( \frac{1}{4}\pi + \frac{1}{2}u \right) \right| + C$$

$$64. \int \operatorname{cosec} u du = \ln |\operatorname{cosec} u - \operatorname{cotg} u| + C = \ln \left| \operatorname{tg} \frac{1}{2}u \right| + C$$

$$65. \int \sec^2 u du = \operatorname{tg} u + C$$

$$66. \int \operatorname{cosec}^2 u du = -\operatorname{cotg} u + C$$

$$67. \int \sec u \operatorname{tg} u du = \sec u + C$$

$$68. \int \operatorname{cosec} u \operatorname{cotg} u du = -\operatorname{cosec} u + C$$

$$69. \int \operatorname{sen}^2 u du = \frac{1}{2}u - \frac{1}{4} \operatorname{sen} 2u + C$$

$$70. \int \cos^2 u du = \frac{1}{2}u + \frac{1}{4} \operatorname{sen} 2u + C$$

$$71. \int \operatorname{tg}^2 u du = \operatorname{tg} u - u + C$$

$$72. \int \operatorname{cotg}^2 u du = -\operatorname{cotg} u - u + C$$

$$73. \int \operatorname{sen}^n u du = -\frac{1}{n} \operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n} \int \operatorname{sen}^{n-2} u du$$

$$74. \int \cos^n u du = \frac{1}{n} \cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$75. \int \operatorname{tg}^n u du = \frac{1}{n-1} \operatorname{tg}^{n-1} u - \int \operatorname{tg}^{n-2} u du$$

$$76. \int \operatorname{cotg}^n u du = -\frac{1}{n-1} \operatorname{cotg}^{n-1} u - \int \operatorname{cotg}^{n-2} u du$$

$$77. \int \sec^n u du = \frac{1}{n-1} \sec^{n-2} u \operatorname{tg} u + \frac{n-2}{n-1} \int \sec^{n-2} u du$$

$$78. \int \operatorname{cosec}^n u du = -\frac{1}{n-1} \operatorname{cosec}^{n-2} u \operatorname{cotg} u + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} u du$$

$$79. \int \operatorname{sen} mu \operatorname{sen} nu du = -\frac{\operatorname{sen}(m+n)u}{2(m+n)} + \frac{\operatorname{sen}(m-n)u}{2(m-n)} + C$$

$$80. \int \cos mu \cos nu du = \frac{\operatorname{sen}(m+n)u}{2(m+n)} + \frac{\operatorname{sen}(m-n)u}{2(m-n)} + C$$

$$81. \int \operatorname{sen} mu \cos nu du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$$

$$82. \int u \operatorname{sen} u du = \operatorname{sen} u - u \cos u + C$$

83.  $\int u \cos u \, du = \cos u + u \sin u + C$

84.  $\int u^2 \sin u \, du = 2u \sin u + (2 - u^2) \cos u + C$

85.  $\int u^2 \cos u \, du = 2u \cos u + (u^2 - 2) \sin u + C$

86.  $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

87.  $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

88. 
$$\int \sin^m u \cos^n u \, du = -\frac{\sin^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} u \cos^n u \, du$$

$$= \frac{\sin^{m+1} u \cos^{n-1} u}{m+n} + \frac{n-1}{m+n} \int \sin^m u \cos^{n-2} u \, du$$

**Formas Contendo Funções Trigonômicas Inversas**

89.  $\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$

90.  $\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

91.  $\int \operatorname{tg}^{-1} u \, du = u \operatorname{tg}^{-1} u - \ln \sqrt{1+u^2} + C$

92.  $\int \operatorname{cotg}^{-1} u \, du = u \operatorname{cotg}^{-1} u + \ln \sqrt{1+u^2} + C$

93. 
$$\int \sec^{-1} u \, du = u \sec^{-1} u - \ln |u + \sqrt{u^2-1}| + C$$

$$= u \sec^{-1} u - \operatorname{cosh}^{-1} u + C$$

94. 
$$\int \operatorname{cosec}^{-1} u \, du = u \operatorname{cosec}^{-1} u + \ln |u + \sqrt{u^2-1}| + C$$

$$= u \operatorname{cosec}^{-1} u + \operatorname{cosh}^{-1} u + C$$

**Formas Contendo Funções Exponenciais e Logarítmicas**

95.  $\int e^u \, du = e^u + C$

96.  $\int a^u \, du = \frac{a^u}{\ln a} + C$

97.  $\int u e^u \, du = e^u(u-1) + C$

98.  $\int u^n e^u \, du = u^n e^u - n \int u^{n-1} e^u \, du$

99.  $\int u^n a^u \, du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a} \int u^{n-1} a^u \, du + C$

100.  $\int \frac{e^u \, du}{u^n} = -\frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1} \int \frac{e^u \, du}{u^{n-1}}$

101.  $\int \frac{a^u \, du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1} \int \frac{a^u \, du}{u^{n-1}}$

102.  $\int \ln u \, du = u \ln u - u + C$

103.  $\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$

104.  $\int \frac{du}{u \ln u} = \ln |\ln u| + C$

105.  $\int e^{au} \sin nu \, du = \frac{e^{au}}{a^2 + n^2} (a \sin nu - n \cos nu) + C$

106.  $\int e^{au} \cos nu \, du = \frac{e^{au}}{a^2 + n^2} (a \cos nu + n \sin nu) + C$

**Formas Contendo Funções Hiperbólicas**

107.  $\int \sinh u \, du = \cosh u + C$

108.  $\int \cosh u \, du = \sinh u + C$

109.  $\int \operatorname{tgh} u \, du = \ln |\cosh u| + C$

110.  $\int \operatorname{cotgh} u \, du = \ln |\sinh u| + C$

111.  $\int \operatorname{sech} u \, du = \operatorname{tg}^{-1}(\sinh u) + C$

112.  $\int \operatorname{cosech} u \, du = \ln |\operatorname{tgh} \frac{1}{2} u| + C$

113.  $\int \operatorname{sech}^2 u \, du = \operatorname{tgh} u + C$

114.  $\int \operatorname{cosech}^2 u \, du = -\operatorname{cotgh} u + C$

115.  $\int \operatorname{sech} u \operatorname{tgh} u \, du = -\operatorname{sech} u + C$

116.  $\int \operatorname{cosech} u \operatorname{cotgh} u \, du = -\operatorname{cosech} u + C$

117.  $\int \sinh^2 u \, du = \frac{1}{4} \sinh 2u - \frac{1}{2} u + C$

118.  $\int \cosh^2 u \, du = \frac{1}{4} \sinh 2u + \frac{1}{2} u + C$

119.  $\int \operatorname{tgh}^2 u \, du = u - \operatorname{tgh} u + C$

120.  $\int \operatorname{cotgh}^2 u \, du = u - \operatorname{cotgh} u + C$

121.  $\int u \sinh u \, du = u \cosh u - \sinh u + C$

122.  $\int u \cosh u \, du = u \sinh u - \cosh u + C$

123.  $\int e^{au} \sinh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \sinh nu - n \cosh nu) + C$

124.  $\int e^{au} \cosh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \cosh nu - n \sinh nu) + C$